'Learning for a fuller life...'

Headteacher: Paul Stanley

Taverham VC CE Junior School: Calculation Guidance 2014

Progression in calculation:

This document details the progression in informal and formal calculations appropriate for each year group, as outlined in the New National Curriculum for 2014. It also recommends the informal methods of calculation to ensure children's mathematical understanding of written processes.

It states in **The national curriculum in England;Key stages 1 and 2 framework document(September 2013)**

The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study. In addition, schools can introduce key stage content during an earlier key stage, if appropriate.

It also states

The expectation is that the majority of pupils will move through the programmes of study at broadly the same pace. However, decisions about when to progress should always be based on the security of pupils' understanding and their readiness to progress to the next stage. Pupils who grasp concepts rapidly should be challenged through being offered rich and sophisticated problems before any acceleration through new content. Those who are not sufficiently fluent with earlier material should consolidate their understanding, including through additional practice, before moving on.

Therefore when using this guidance staff and parents should remember children's fluency and understanding in calculation should be given greater emphasis than their ability to use an informal or formal written method (stage before age). An **appendix** of what we expect children to have learnt in Y1 and Y2 is included at the end.

Strategies:
The strategies that children should be aware of are detailed for each year group. Children should be encouraged to use a range of strategies and to consider the most appropriate strategy for any given calculation. Children's ability to consider a range of strategies should be given greater emphasis than their ability to use a particular informal or formal written method.

Models and Images:
At all stages of calculation, children need experience of concrete materials and to understand the connection between the mathematical processes and the written forms of calculation.

Problem Solving:
The National Curriculum for Mathematics aims to ensure that all pupils can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

Problem solving should be a key element which is interwoven into maths lessons throughout the year groups and to help embed the learning of calculations. Children should be given a variety of rich problems to increase their fluency and understanding at a particular stage.

Rationale for using written methods
1. To make clear a mental procedures for the pupil.
2. To help communicate methods and solutions.
3. To provide a record of work to be done.
4. To ensure accuracy of calculation when the problem is too difficult to be done mentally.
5. To develop and refine a set of rules for calculation.

Readiness for informal and formal calculations (This is only a guide)

When are children ready for written calculations?
Addition and subtraction:
a. Do they know addition and subtraction facts to 20?
b. Do they understand place value and can they partition numbers?
c. Do they understand the relationship between addition and subtraction?
d. Can they explain their mental strategies orally and record them using informal jottings?

Multiplication and division:
a. Do they know the 2, 3, 4, 5, and 10 times tables?
b. Do they know the result of multiplying by 0 and 1?
c. Do they understand 0 as a place holder?
d. Can they multiply two and three digit numbers by 10 and 100?
e. Can they double and halve two digit numbers mentally?

Addition and Subtraction
Year 3

Progression in Calculation:
- add and subtract numbers mentally, including:

  a three-digit number and 1s.
  a three-digit number and 10s.
  a three-digit number and 100s.

- add and subtract numbers progressing to 3 digits, using a written method.

- estimate the answer to a calculation and use inverse operations to check answers.
- solve varied problems, including missing number problems, using number facts, place value and more complex addition and subtraction.
- pupils use their understanding of place value and partitioning, to use the columnar method for addition and subtraction for 2 digit numbers, (increasing to 3 digits) **by the end of the year**.
- answers to mental calculations can exceed 100.

Written strategies:

Addition
Informal methods

- use a number line or hundred square to support calculation: e.g. 35 + 23 =
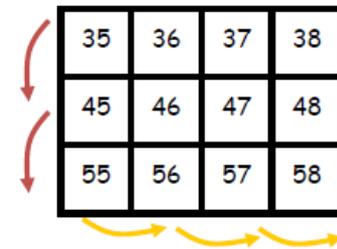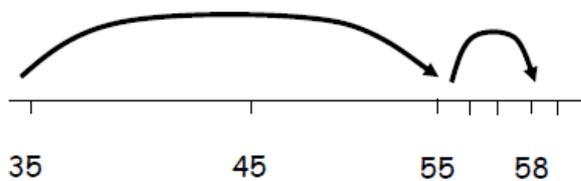
- partition numbers (using place value cards/ apparatus) and make efficient jottings with and without number lines.

24+17
20+10 =30   4+7 = 11   ⟶   30+11 = 41

- the expectation is that Year 3 children will be able to bridge through 10, however teachers need to ensure place value is secure before moving to this stage.

Formal methods

- Expanded column method.

$$
\begin{array}{r}
48 \\
+\ 36 \\
\hline
40 + 8 \\
30 + 6 \\
\hline
70 + 14 = 84
\end{array}
$$

Note: the use of different colour for hundreds/tens/units can help children understand the algorithm.

- Compact column method

```
   4 8
+  3 6
   ___
   8 4
   ___
   1
```

Subtraction
Informal methods
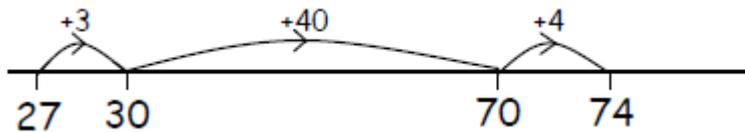
- partitions numbers (using place value cards/ apparatus) and make efficient jottings with and without number lines and with no need to exchange.

37-16
16 = 10+ 6 $\longrightarrow$ 37-10 =27   27 - 6 = 21

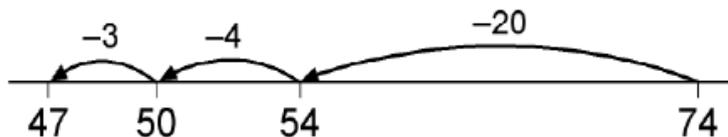- use a number line or hundred square to support calculation: e.g.

Adding on method

74-27 = 47

+3   +40   +4

27 30   70 74

Counting back method

74 – 47 = 27

−3   −4   −20

47   50   54   74

Formal methods

- Expanded column method (no need for exchange).

(no need for exchange)
48
- 36
40 + 8
30 + 6
10 + 2 = 12

(need for exchange)
52
- 36
$^{40}\cancel{50}^{-10}$ + 2^{+10}
30 + 6
10 + 6 = 16

- Compact column method

(no need for exchange)
6 8
- 3 5
3 3

(need for exchange)
$^4\cancel{5}$ ^{1}2
- 2 5
2 7

Year 4

Progression in Calculation:
- find 1000 more or less than a given number.
- add and subtract numbers with up to **4 digits** using the formal written methods of columnar addition and subtraction where appropriate. Ensure that children are exposed to calculations involving numbers with varying numbers of digits (e.g. 453 + 72).
- estimate and use inverse operations to check answers to a calculation.
- solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why. This should include decimals within the context of money

  Also
- add and subtract fractions with the same denominator.

Written Strategies (use models and images as for Year 3):

- refine expanded methods of column addition and subtraction. Making sure that the link between the expanded method and the compact method are explicit. It is still very important that concrete materials are used for visual learners. (see Y3)

- children need to make appropriate decisions when deciding whether to calculate mentally or to use a written calculation.

    For example: 2002 – 1999 would be most efficiently solved using a mental method.

- Adding and subtracting fractions with the same denominator (show we only add numerator):

$$\frac{1}{4} \quad + \quad \frac{2}{4} \quad = \quad \frac{3}{4}$$

Year 5

Progression in Calculation:

- add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction). Ensure that children are exposed to calculations involving numbers with varying numbers of digits (e.g. 1453 + 72).
- add and subtract numbers mentally with increasingly large numbers.
- use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy.
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.
- add and subtract fractions with the same denominator and with denominators that are multiples of the same number (e.g. 1/3 + 2/9).
- practise mental calculations with increasingly large numbers to aid fluency (for example, 12,462 – 2,300 = 10,162).
- practise adding and subtracting decimals, including a mix of whole numbers and decimals, decimals with different numbers of decimal places and complements of 1 (e.g. 0.83 + 0.17 = 1).

Written Strategies:

- refine compact methods of column addition and subtraction(see Y3), moving to decimals(of same number of decimal places).

$$
\begin{array}{r}
2\ 6\ 7\ .\ 7 \\
+\ 3\ 5\ 1\ .\ 4 \\
\hline
6\ 1\ 9\ .\ 1 \\
{\scriptstyle 1 \quad\quad 1}
\end{array}
$$

$$
\begin{array}{r}
5\ {}^{5}\!6\ .\ {}^{1}2 \\
-\ \ 2\ 3\ .\ 5 \\
\hline
3\ 2\ .\ 7
\end{array}
$$

- Find common denominators when adding and subtracting fractions

$$
\frac{7}{8} \quad - \quad \frac{1}{4} \quad = \quad \frac{7}{8} \quad - \quad \frac{2}{8} \quad = \quad \frac{5}{8}
$$

Year 6

Progression in Calculation:

- perform mental calculations, including with mixed operations and large numbers
- use their knowledge of the order of operations to carry out calculations involving the 4 operations.
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.
- solve multi-step problems involving addition and subtraction.
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.
- add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions.
- pupils should practise, use and understand the addition and subtraction of fractions with different denominators by identifying equivalent fractions with the same denominator. They should start with fractions where the denominator of one fraction is a multiple of the other and progress to varied and increasingly complex problems.

Written Strategies:

- refine compact methods of column addition and subtraction including using decimals with different decimal places**.**

$366.7 + 462.52 =$

$$
\begin{array}{r}
3\,6\,6\,.\,7\,0 \\
+\ 4\,6\,2\,.\,5\,2 \\
\hline
8\,2\,9\,.\,2\,2 \\
\end{array}
$$
$$\quad\ 1 \qquad 1$$

$86.26 - 34.8 =$

$$
\begin{array}{r}
8\ {}^{5}\!6\,.\,{}^{1}2\,6 \\
-\ \ \ 3\,4\,.\,8\,0 \\
\hline
5\,1\,.\,4\,6 \\
\end{array}
$$

- children to move onto adding and subtracting fractions by finding common denominators, teachers to extend the children to adding and subtracting fractions where neither are multiples of each other.

Addition example:

$$\frac{3}{5}\ (\times 3) + \frac{4}{15} = \Box$$
$$\frac{9}{15} + \frac{4}{15} = \frac{13}{15}$$

$$\frac{3}{4}\ (\times 3) + \frac{1}{3}\ (\times 4) = \Box$$
$$\frac{9}{12} + \frac{4}{12} = \frac{13}{12} = 1\frac{1}{12}$$

Multiplication and Division

Year 3

Progression in Calculation:
- find 10 lots or 100 lots more or less than a given number.

- recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables.

- pupils continue to practise their mental recall of multiplication tables when they are calculating mathematical statements in order to improve fluency. Through doubling, they connect the 2, 4 and 8 multiplication tables.

- pupils develop efficient mental methods, for example, using commutativity and associativity (for example, $4 \times 12 \times 5 = 4 \times 5 \times 12 = 20 \times 12 = 240$) and multiplication and division facts (for example, using $3 \times 2 = 6$, $6 \div 3 = 2$ and $2 = 6 \div 3$) to derive related facts ($30 \times 2 = 60$, $60 \div 3 = 20$ and $20 = 60 \div 3$).

- write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods (grid method for multiplication).

- solve problems, including missing number problems, involving multiplication and division.

Written Strategies for Multiplication:

Informal Written Methods
- use practical strategies and apparatus to solve problems.

E.g. 4 x 13 =
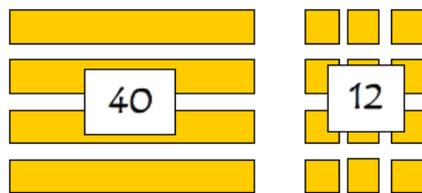
40 + 12 = 52

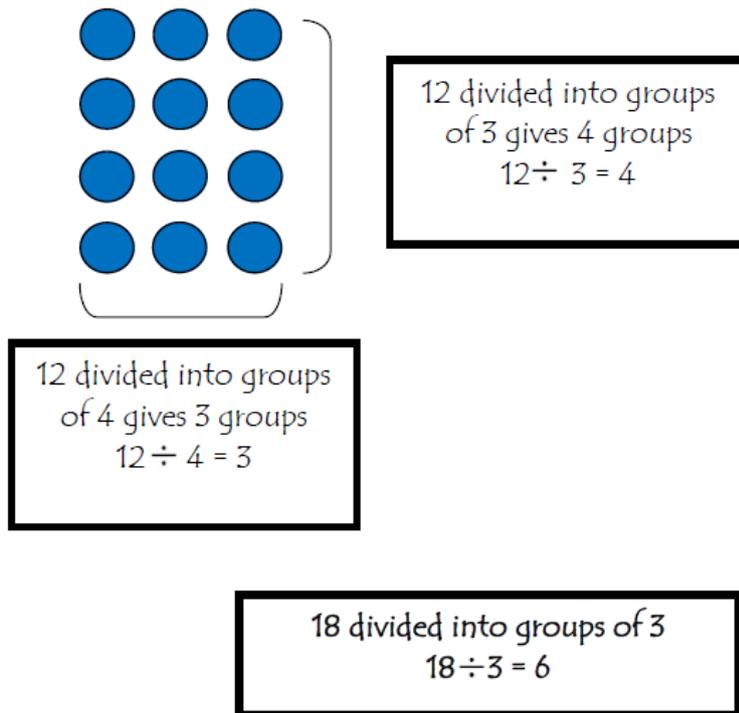**Formal Written Methods**
- solve **u x tu** using grid method

| x	20	3
4	80	12

80 + 12 = 92

Written Strategies for Division:

Informal Written Methods

- look at the connections between multiplication and division and use the inverse to solve division problems with their knowledge of multiplication. This can be done with concrete materials or making jottings.

12 divided into groups
of 3 gives 4 groups
$12 \div 3 = 4$

12 divided into groups
of 4 gives 3 groups
$12 \div 4 = 3$

18 divided into groups of 3
$18 \div 3 = 6$

- represent 'groups' for division on a number line using apparatus alongside the line

3   6   9   12   15   18

$18 \div 3 = 6$
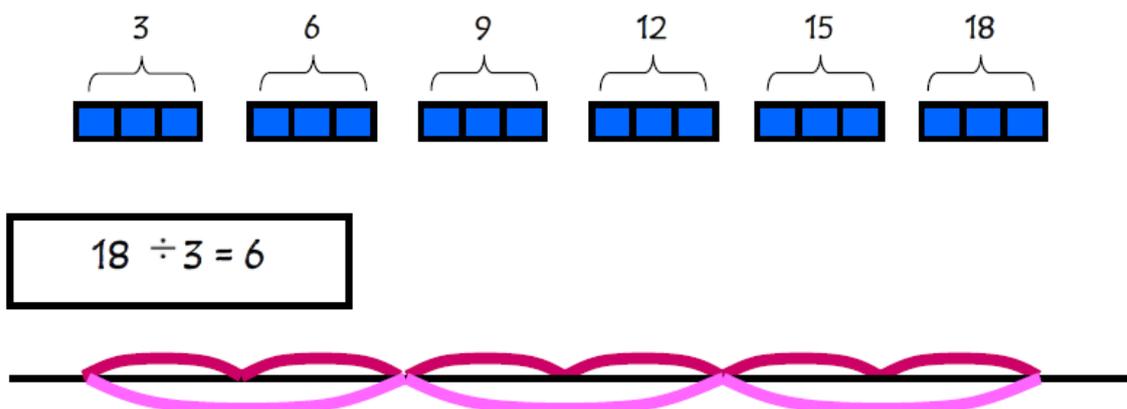
Year 4

Progression in Calculation:

- recall multiplication and division facts for multiplication tables up to 12 × 12.
- pupils continue to practise recalling and using multiplication tables and related division facts to aid fluency.
- pupils practise mental methods and extend this to 3-digit numbers to derive facts, (for example 600 ÷ 3 = 200 can be derived from 2 x 3 = 6).
- recognise and use factor pairs and commutativity in mental calculations.
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout.
- pupils practise to the formal written method of short multiplication solve problems.
- involving multiplying and adding in contexts, including using the distributive law to multiply two-digit numbers by 1 digit.

Written Strategies:
Multiplication

Formal Written Methods

- refine the use of the formal written method to multiply two and three digit numbers by one digit**(HT x U, HTU x U)** and start to show links to short multiplication.

E.g. 23 x 4 =

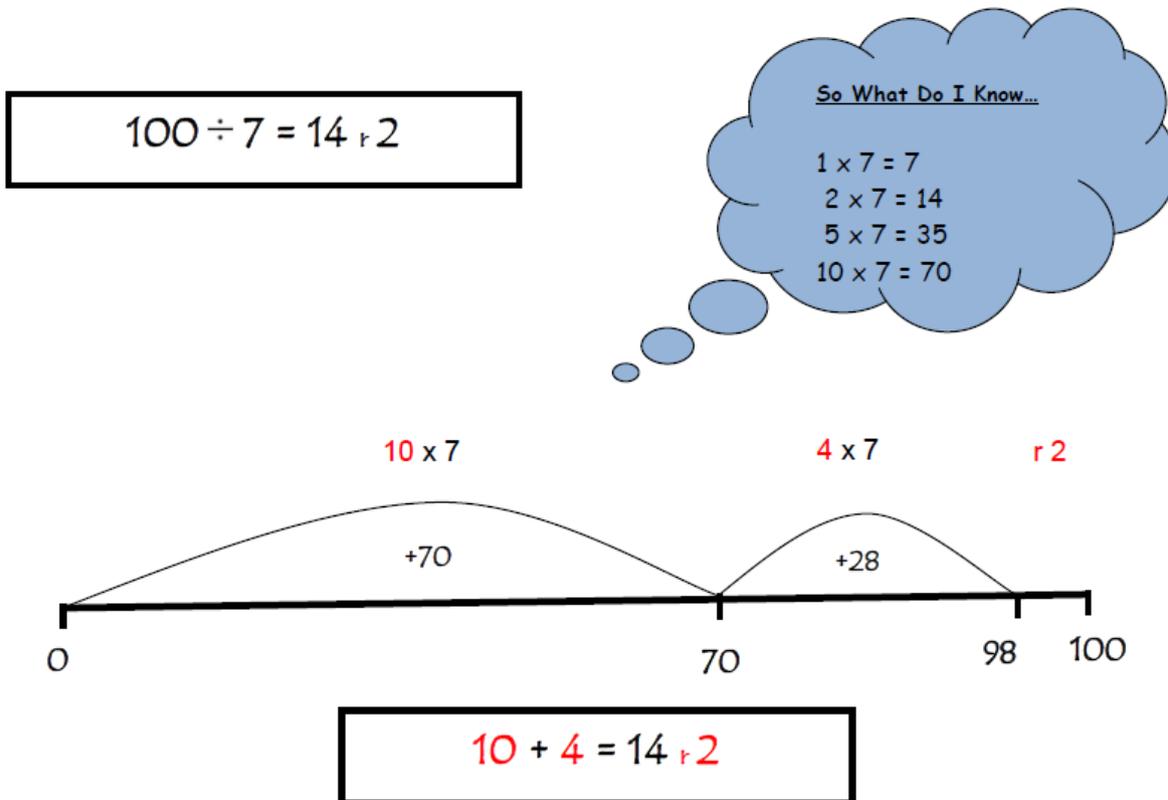Ensure the children are aware of the link between the grid method and the formal written method.

×	20	3
4	80	12

80 + 12 = 92

```
  23
x  4
-----
  92
   1
```

Division

Informal Written Methods

- divide two and three digit numbers by single digits using a number line. Use a 'so what do I know' box to divide by larger numbers.

$$100 \div 7 = 14 \text{ r} 2$$

So What Do I Know...

$1 \times 7 = 7$
$2 \times 7 = 14$
$5 \times 7 = 35$
$10 \times 7 = 70$

10 x 7   4 x 7   r 2

+70   +28

0   70   98   100

$$10 + 4 = 14 \text{ r} 2$$

Start at zero and work towards the target number by adding.

Year 5

Progression in Calculation:
- multiply numbers up to 4 digits by a one or two-digit number using a formal written method, including long multiplication for two-digit numbers.
- multiply and divide numbers mentally drawing upon known facts.
- divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.
- multiply and divide whole numbers and those involving decimals by 10, 100 and 1000
- solve problems involving multiplication and division
- Pupils interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as decimals or by rounding (for example, 98 ÷ 4 = 24 r2 = 24.5 = 25).

Written Strategies:
Multiplication

- use grid method for **tu x tu, HTU x U** and **ThHTU x U**

×	30	3
10	300	30
4	120	12

= 330
= +132
—————
462

```
 300
 120
  30
+ 12
————
 462
```

- use formal written method to multiply **TU x TU** progressing to **HTU x TU** and **ThHTU x TU**. Ensure that the children see the link between the grid method and the formal written method.

56 x 27 =

```
    56
 x  27
 —————
   392   (56 x 7)
  1120   (56 x 20)
 —————
  1512
```

Ensure children are aware of and understand the role of zero as a place holder.

Division

- use the formal written method (bus stop method/ short division) for division. Children should be dividing upto four digit numbers by single digits.

- children to be giving answers with remainders, as decimals and by using rounding.

- children need to interpret how to answer a division question considering the context of a problem.

$$5715 \div 6 =$$

Answer with a remainder:

$$6 \overline{)5\,^5 7\,^3 1\,^1 5} = 0\,9\,5\,2\,\text{r}3$$

Answer as a decimal:

$$6 \overline{)5\,^5 7\,^3 1\,^1 5\,.\,^3 0} = 0\,9\,5\,2\,.\,5$$

Answer rounded:

953

Progression in Calculation:

- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication.
- divide numbers up to 4 digits by a two-digit whole number using either the formal written method of long division or short division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context.
- interpret remainders according to the context.
- perform mental calculations, including with mixed operations and large numbers.
- solve problems involving addition, subtraction, multiplication and division.
- multiply simple pairs of proper fractions, writing the answer in its simplest form (for example, ½ x ½ = ¼)
- multiply one-digit numbers with up to two decimal places by whole numbers.
- use written division methods in cases where the answer has up to two decimal places.

Written Strategies

Multiplication

- refine formal written method of multiplication(Y4 and Y5), progressing to larger numbers.

- progress to multiplying decimal numbers(upto 2dp) by whole numbers by multiplying by 10 or 100 first and then dividing by 10 or 100 at the end.
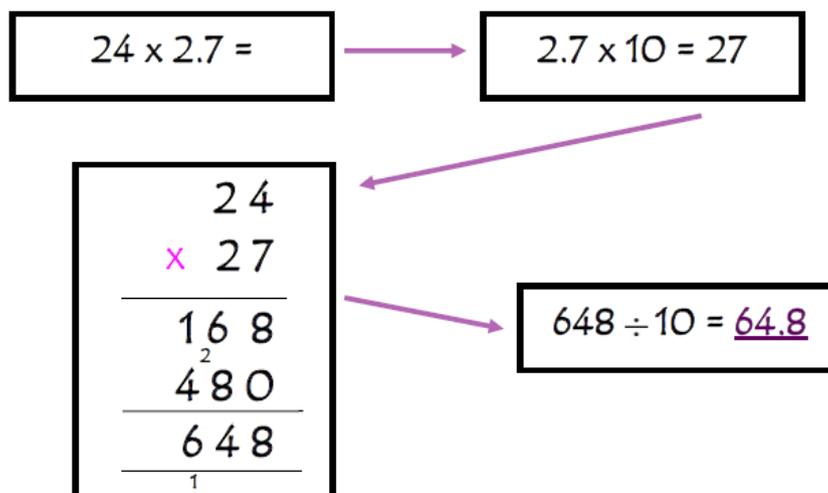
- when multiplying fractions children should be taught to multiply the numerators, the denominators and then look to simplify the answer if possible

Multiplying Fractions

$$\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$$

$$\frac{2}{3} \times \frac{1}{4} = \frac{2}{12} = \frac{1}{6}$$

Division

- refine formal written method for division(Y5), progressing to dividing by two digit numbers.

- children to decide, dependent on the calculation whether, the long or short written method would be most appropriate.

$$432 \div 15 =$$

Short Division:

$$028 \text{ r}12$$
$$15 \overline{\smash{\big)}\, 4\,^{4}3\,^{13}2}$$

So What Do I Know...

1 x 15 = 15
2 x 15 = 30
4 x 15 = 60
5 x 15 = 75
10 x 15 = 150

Long Division

$$028 \text{ r}12$$
$$15 \overline{\smash{\big)}\, 4\,3\,2}$$
$$\,3\,0\,0 \quad (20 \times 15)$$
$$\overline{1\,3\,2}$$
$$\,1\,2\,0 \quad (8 \times 15)$$
$$\overline{\,1\,2}$$

APPENDIX: Expected Year 1 and 2 Progression in Calculations

Addition and Subtraction
Year 1:

Progression in Calculation:

- given a number, identify one more and one less.
- identify and represent numbers using objects and pictorial representations including the number line, and use the language of: equal to, more than, less than (fewer), most, least.
- represent and use number bonds and related subtraction facts within 20.
- add and subtract one-digit and two-digit numbers to 20, including zero.
- solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems.
- pupils memorise and reason with number bonds to 10 and 20 in several forms. They should realise the effect of adding or subtracting zero. This establishes addition and subtraction as related operations.
- pupils combine and increase numbers, counting forwards and backwards. They discuss and solve problems in familiar practical contexts, including using quantities.

Multiplication and Division
Year 1:

Progression in Calculation:

- solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.
- Through grouping and sharing small quantities, pupils begin to understand: multiplication and division; doubling numbers and quantities; and finding simple fractions of objects, numbers and quantities.
- They make connections between arrays, number patterns, and counting in twos, fives and tens.

Addition and Subtraction
Year 2:

Progression in Calculation:

- solve problems with addition and subtraction, using concrete objects and pictorial representations, including those involving numbers, quantities and measures.
- recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100.
- add and subtract numbers using concrete objects, pictorial representations, and mentally, including:

  -a two-digit number and ones.
  -a two-digit number and tens.
  -two two-digit numbers.
  -adding three one-digit numbers.
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot.
- recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.

Multiplication and Division
Year 2:

Progression in Calculation:

- Pupils are introduced to the multiplication tables. They practise and become fluent in the 2, 5 and 10 multiplication tables.
- recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.
- calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs.
- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.
- solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.